Strong Interaction
part 1

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February - March 2025

1 Abstract

The aim is to explore the strong interaction from an elementary physics perspective. We have learned about the atomic nuclei and its composition back in high school and now we further our exploration into the standard model of particle physics. Looking at the standard model, it is necessary to understand the proton and the neutron , the basic composition of which is formed by quarks. Quarks exchange gluons which are spin 1 particles that mediate the strong interaction. The field that studies quarks and gluons and how they facilitate strong interaction is called QCD or Quantum chromodynamics. The three colors of quarks , red, green and blue is why have we the name chromo. Just like electric charge is conserved for spin 1/2 particles, similarly for quarks color charge is conserved(It is not analogous to electric charge). We move onto phases of matter of QCD, starting from symmetry breaking in the early universe to hadronization(confinement). We also take a deep dive into learning about a few quantum numbers like isospin, strangeness, charge and how they conserve strong interactions and overall flavor dynamics and flavor symmetry. Our investigation continues into how meson pairings are formed and hadron nomenclature on the basis of isospin. Major focus has been devoted towards understanding how isospin conserves strong interaction and by virtue of symmetry(strangenss vs charge in our case) come up with a way of classifying mesons and baryons.

2 Introduction

As it has been known since childhood, the atomic nuclei consists of neutrons and protons with electrons orbiting around it. The neutron has a neutral 0 charge , while the proton has a positive +1 charge. In fact, the stability of an element depends on its neutron number. As time went on , it had been discovered that the neutrons and protons were made up of something more fundamental , known as quarks. The neutron consists of 3 quarks -¿ two down quarks, one up quark and the proton -¿ two up quarks and a down quark.

Now, each of these quarks have fractional charge unlike the electron, proton and neutron. Up quark has a $+2/3e$ charge while the Down quark has $-2/3e$ charge. Since, the quarks have $1/2$ spin , these are classified into fermions in the standard model of particle physics.

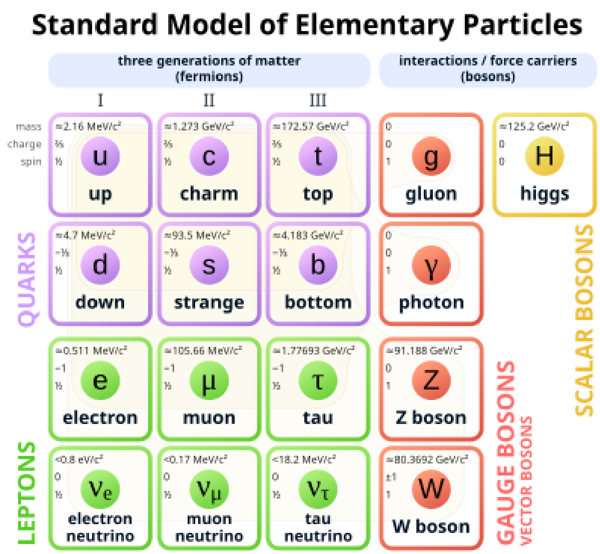

Figure 1: standard model, source:wiki

	Charge	First Generation	Second Generation	Third Generation
Quarks	$+2/3$	up (u)	charm (c)	top (t)
	$-1/3$	down (d)	strange (s)	bottom (b)
Leptons	-1	electron (e)	muon (μ)	tau (τ)
	0	electron neutrino (ν_e)	muon neutrino (ν_μ)	tau neutrino (ν_τ)

Figure 2: source: Research gate

Particles in the standard model are divided into generations (quantum number and mass). For the six quarks, each generation has two quarks each.
1st generation- Up and down, 2nd generation- strange($-1/3e$) and charm($+1/3e$), 3rd generation- top($+2/3e$) and bottom($-1/3e$).
A combination of two or more quarks are known as hadrons.
One might ask, how come the neutrons and the protons inside the nuclei are held together without them collapsing onto each other and becoming some kind of a particle soup ?. The answer is the strong force. The strong interaction is responsible for holding the neutrons and protons together. One of the four fundamental forces in our universe is the strong force. The others being the electromagnetic force, the weak force and gravity and the relationship between the electromagnetic force and gravity is still unknown to us!.

Now, if the strong force holds the neutrons and protons together, there must be some kind of a particle exchange that happens between them that mediates this kind of interaction. Discovered by Murray Gell-mann in the 60s, these particles are called gluons. You can think of them as the glue that holds the particles together.
Gluons act as a force mediator between the quarks. They have a spin of 1. The quarks exchange gluons between them. We will learn more about the quarks and the gluons a little later. Quarks and gluons together form hadrons as mentioned earlier.

3 Phases of Matter

If we start from the early universe, moments after the Big Bang, for about 10^{-46} seconds , gravity was separated and was the most dominating force. The other fundamental forces were condensed into a grand unified force. Around 10^{-36} seconds, inflation happens and the universe expands radically. The temperature of the universe was still too high at this point and the quarks remained together as hadrons, also known as quark epoch. Around 10^{-12} seconds after the big bang, the electroweak interaction ends as the weak force and the electromagnetic force become separated. Now, the weak nuclear force is a short range force, that can interact with the higgs field and provide mass to the W and Z bosons(Symmetry breaking happens here). The quark epoch still exists as the temperatures are too high at this point. At about 10^{-6} seconds, the temperature of the universe has finally cooled down to about 10^{12} Kelvin and the quark-hadron transition takes place, quark confinement happens. Quarks become confined into hadrons. 3 to 20 minutes after the big bang, the nuclei has formed and about $380,000$ years later, the atoms are formed.

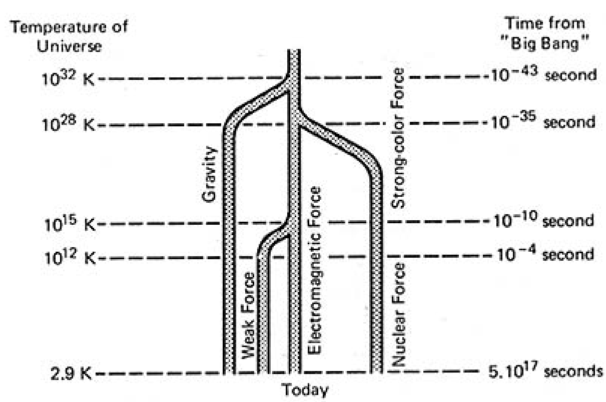

Figure 3: Force seperation in early universe, symmetry breaking, source: david ling

Theoretically, a lot of work has been put over the years to understand phase transitions in QCD.

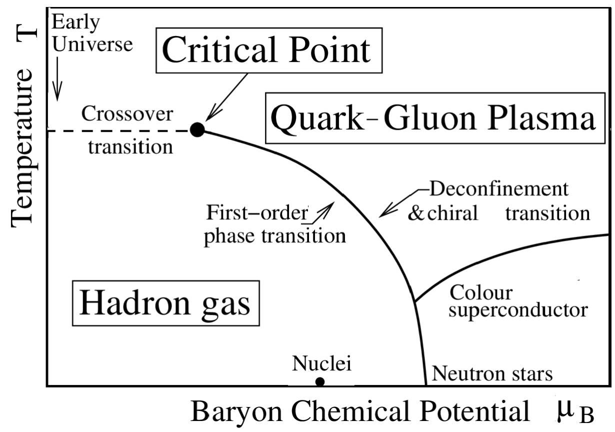

Figure 4: Phase transition diagram in QCD, source: wiki

Think about the diagrammatic representation above and divide it into four quadrants. The phase transition line divides the three quadrants horizontally from the middle.
- Quadrant 1)- Low potential, low temp.
- Quadrant 2) - high potential, low temp.
- Quadrant 3) - high potential, high temp.

For the 1st quadrant- At low temperature and low potential, we have ordinary atomic matter surrounded by vacuum. At low temperatures there isn't any presence of antiquarks and as mentioned above, we are in a state of confinement.

As we move towards the 2nd quadrant, by increasing the potential and at low temperatures, neutron stars are formed. As we increase the potential, the quark density also increases and we move into a phase of highly compressed nuclear matter. Further increase in potential results in color superconductivity. This is where baryon densities are so high, that the color charge becomes directly analogous to charge(current).

When we try to heat up the system and move away from confinement towards the 3rd quadrant, at around 150-170 MeV confinement breaks away the hadron composition. Thermal fluctuations break up the pions and we move into a region of the early universe where there are gases made up of quarks, anti-quarks, gluons etc. The quark epoch persists here. At about 170 MeV , as we are moving along the line of phase transition, we encounter a critical point. This point is believed to be where symmetry breaks. This is the boundary between confined and unconfined phases.

4 Flavour Dynamics

As is mentioned above, the particles in the standard model, are divided into generations. For the three generations, each generation has two quarks each. We need to talk about quantum numbers here, because the conservation of such quantum numbers are essential in describing the behavior of the baryons.

The five flavours(which represents the species of the elementary particles, SU(2)) can describe the quantum state of the quarks and as a result multiple quarks can form(mesons, hadrons and baryons). These are as follows:-

- Isospin
- strangeness
- charm
- topness
- bottomness

As per conservation laws, charge is conserved. The conserved quantum numbers in strong interaction are:- electric charge(Q), the third component of isospin(T3) , baryon number(B) and Lepton number (L).

To be a little bit more precise, the eigenvalues of charge is conserved. The flavour states can also undergo superposition among each other. It should be noted that spin co-ordinates in this case, should not be confused with intrinsic spin or angular momentum. Isospin in this case is related to the up-down quark content.

Isospin Invariance:

If we take a look at the mass of the Neutron and the Proton, to a good approximation they have more or less the same mass (938 Mev). For this reason, they can be interpretated as two states of the same particle. The component $|I_3\rangle$ of the operator $\hat{T_3}$ has the eigen values $-1/2$ and $+1/2$. This is related to the charge operator $\hat{Q}$. For a system of n - nucleons the charge operator depends on the mass number. The Hamiltonian H of the strong interaction remains isospin invariant.

If we consider it quantum mechanically, upon interaction, the isospin eigenstates should be invariant $H|I_{int}\rangle = I_{int}|I_{int}\rangle$.

We are going to be proving it in details down the line.
The geometric representation of isospin is a 2 to 1 mapping of SO(3, 1)
For a given isospin quantum number I, $2I + 1$ states are allowed, which are known as isospin multiplets.

Co-ordinate Components

For isospin $I(I_x, I_y, I_z)$, I is vector valued with 3 components. For up and down quarks , I has a value of $+1/2$, with a 3rd component I_3 being $+1/2$ for up and $-1/2$ for down quarks. We should consider up vs down as a projection onto the z-axis.

$I_3 = 1/2(n_u - n_d)$ where n_u and n_d are the number for the up and down quarks respectively.

5 Meson Pairings

Going further deep into the interaction between the quarks, we realise that Pauli's exclusion principle does not apply to bosons. Therefore, each quark must have a different color as a means to distinguish one quark from another. The question now is that, how does color conversion happens between the quarks ?

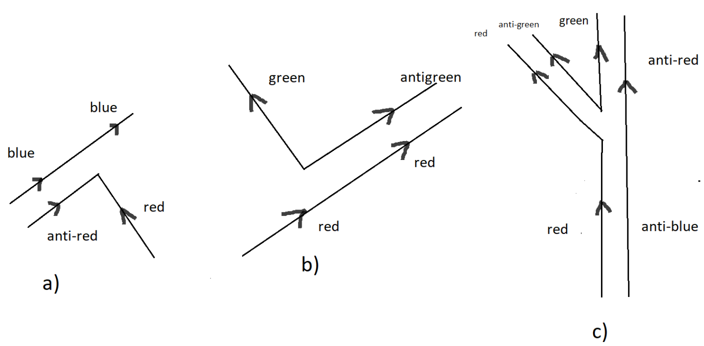

Figure 5: color conversion within quarks

In figure b) we can see that a red quark gets convered into a green quark with a red anti-green gluon pairing. Since, we know that gluons hold the quarks together, they also assist in a color change, either by absorption or emission. This is exactly what is happening in the figure a). A blue-antired pairing comes in, a red quark comes in and a blue quark goes out.
Gluons can also make other gluons, as seen in figure c).

One thing that needs to be noted is that quarks cannot be isolated and cannot be observed below 130-140 Mev.
Suppose , a blue quark is about to leave a red, green, blue hadron. It cannot leave without forming a blue-antiblue pairing. This phenomenon is known as

quark confinement.

$rgb- > rgb b\tilde{b} - > rgb + b\tilde{b}$

These pairings are called meson pairings and are colourless. The color force favours confinement and is thus favourable to create a $q\tilde{q}$ meson pairs. Also, quarks and gluons cannot be separated without forming new hadrons.

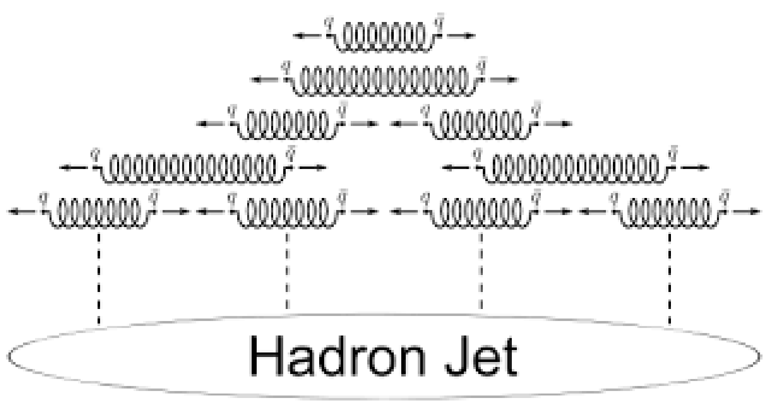

Figure 6: Hadron jet , source:-wiki

The above image represents the functionality of a rubber band, it is favourable to create meson pairing rather rather than elongating the flux.

Further investigation into the forces between the particles were done and Yukawa in the 1930s predicted a particle known as pions. It was predicted that that if you hit the proton against the nucleus, it would knock out such particles and sure enough such particles came out(plenty of other particles also came out like kaons and sigma baryons and lambda baryons etc). Pions as it turns out, also carries the residual nuclear force and this force only lasts upto the width of the nucleus. This process of pion exchange produces the force, that holds the nucleus together.

pions (virtual) get absorbed by neutron and produces more gluons which interact with the quarks to exchange color. This in turn will produce more pions , which will get absorbed by the proton.

6 Gell-Mann Matrices

Because of pion absorption or emission, the gluons are subjected to color change. The Quarks as we know, carry three colors and the antiquarks carry, three anti-colors. For a SU(3) symmetry, possible combinations are listed below.

$r\tilde{r}, r\tilde{g}, r\tilde{b}, b\tilde{r}, b\tilde{g}, b\tilde{b}, g\tilde{r}, g\tilde{b}, g\tilde{g}$
Therefore, it can be said that gluons can be categorized into an eight $(2^3 - 1)$(sum of three diagonal elements must be zero) color states , or octet which are equivalent to the gell-mann matrices.

If we imply a gauge field A_μ ,
the Gell-mann matrices can be written as
$A_\mu = A_\mu^a \lambda^a$, where $a = 1, 2, 3....8$.
These matrices are traceless, Hermitian and a unitary matrix group of SU(3). (3 * 3)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Figure 7: Gell-mann matrices, Source:- semantic scholar

7 Hadrons, Hadron Nomenclature

As mentioned previously, isospin formally represents angular momentum, as more particles were discovered, they were assigned into isospin multiplets, according to different charge states. Based on isospin the particles can be assigned as follows.

- I = 0 -¿ lambda baryons(Λ^0), eta meson(η) . They are singlets.
- I = 1/2 -¿ K-mesons, protons(p) and neutrons(n). They are two doublets.
- I = 1 -¿ sigma baryons(Σ^0, Σ^+, Σ^-). They are triplets.
- I = 3/2 -¿ delta baryons(Δ^{++}, Δ^+, Δ^0, Δ^+). They form quartets.

For baryons isospin is 1/2 and 3/2, meanwhile for mesons isospin is 0 or 1.

The mesons are formed by electron-positron collison.

Another quantum number that indicates symmetry is the total angular momentum or J^p parity. On the basis of parity and spin , mesons and baryons can be further assigned into

Mesons

- scalar mesons ($J^p = 0^+$)
- pseudoscalar mesons ($J^p = 0^-$)
- vector mesons. ($J^p = 1^-$)

Baryons

- Baryon octet ($J^p = +1/2$)
- Baryon decuplet ($J^p = +3/2$)

Some of them are as follows.

pseudocalar mesons ($J^p = 0^-$)

K^0, K^+, K^-, K^{-0}, π^0, π^+, $\pi-$, η.

baryon octet (1/2)

neutron, proton, Σ^0, Σ^+, Σ^-, Λ^0, Ξ^-, Ξ^0

baryon decuplet(3/2)

Δ^{++}, Δ^+, Δ^0, Δ^+, $\Xi^{'-}$, $\Xi^{'+}$, $\Sigma^{'0}$, $\Sigma^{'+}$, $\Sigma^{'-}$, Ω^-

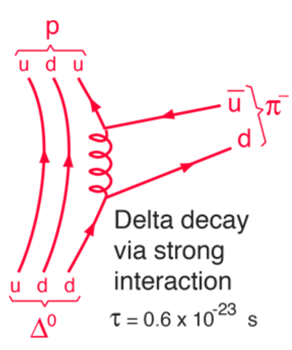

Figure 8: Delta baryon decay, Source:- Hyperphysics

There are many such particles. All of the delta baryons with a mass of about MeV quickly decay via the strong interaction into a nucleon (neutron or proton) and a pion of appropriate charge. Ordinary baryons like the proton or neutron have a mass of about 938 MeV and both have an intrinsic isospin of 1/2 . The Feynman diagram showing a delta baryon decaying into a proton and a pion is shown above.

8 Isospin wave function

In a baryon, the color coupling of the three quarks form a singlet which is antisymmetric under the exchange of three quarks. This is the reason why we have color.

A consequence of quarks being fermions is that the wave-function of a baryon must be antisymmetric under the exchange of two quarks, meaning that the overall state changes sign when two quarks are swapped. We will see it take form down the line.

Isospin wave functions are also formed in the same way, with $\uparrow$ by u and $\downarrow$ by d. States with three identical quarks like Ω^- (sss) have a simple structure. Either the spin wave function corressponds to spin $S = 3/2$ and spin has to be symmetric or one can combine the corresponding spin-wave function with a pair of mixed symmetry wave functions.

When isospin $I = 3/2$, this is the Δ family. When $I = 1/2$, new arrangements exists. So, basically $S = +1/2$ and $I = 1/2$ can be combined to form a symmetric wave function.

So, the aim is to transform an antisymmetric wave function to a symmetric wave function.

9 Flavour Decomposition in Baryons

Flavour decomposition in baryons refer to expressing the baryons quantum states in terms of its constituent quark flavour. The 3 lightest quarks -¿ up(u), down(d), strange(s) form a flavour symmetry. Baryons are classified into multiplets because of this symmetry. Octet(8) and decuplet(10).

Flavour wave-function construction

The total wavefunction consists of flavour, spin, color and spatial components. The Octet(8) has a mixed symmetry wave function, meanwhile the

decuplet(10) has a completely symmetric wave function.
For eg:- the proton (uud) can be expressed in a flavour symmetry form using mixed symmetry

$|p\rangle = 1/\sqrt{2}(|uud\rangle - |duu\rangle)$

while,

$\Delta^+(uud)$ in the decuplet has a fully symmetric form:-

$|\Delta^+\rangle - |uud\rangle$

Now, in terms of quark content

- $\Lambda(uds)$ - anti-symmetric flavour wave function.
- $\Sigma^0(uds)$ and $\Sigma^{+-}(uus, dds)$ have a mixed symmetry wave function.

- Xi baryons follow similar rules.

Heavier quarks like charm(c) and bottom(b) extend the decomposition to SU(4) etc.

Structure of a baryon's wave function

$\psi_{total} = \psi_{flavour} \otimes \psi_{spin} \otimes \psi_{color} \otimes \psi_{spatial}$

- Color wavefunction - totally antisymmetric (singlet)
- Spatial wave function - symmetric in the ground state.
- Flavour spin part - $(\psi_{flavour} \otimes \psi_{spin})$-¿ totally symmetric since color is antisymmetric.

Thus, flavour spin part must be symmetric under quark exchange in the decuplet and mixed symmetric in the octet.

1. **Baryon decuplet spin = -3/2. (Fully symmetric)**

$\psi^{\Delta^+}_{flavour} = |uud\rangle$

Therefore, the total will be

$\psi^{\Delta^+}_{total} = \psi^{\Delta^+}_{flavour} \otimes \psi^{\Delta^+}_{sym-spin} \otimes \psi^{\Delta^+}_{antisym-color}$

2. **Baryon octet (proton (uud)) spin = -1/2 (Mixed symmetry)**

The flavour wave function must be mixed symmetric since the spin wave function is also mixed symmetry.

$\psi^p_{flavour} = 1/\sqrt{2}(|udu\rangle - |duu\rangle)$ (mixed symmetry)

$\psi^p_{spin} = 1/\sqrt{2}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle)$ (mixed symmetry)

Therefore, total wave function will be:-

$\psi^p_{total} = \psi^p_{flavour} \otimes \psi^p_{spin} \otimes \psi^p_{antisym-color}$

Baryon wave-function derivation($J^p = 3/2$) - decuplet

Baryons are color singlets, hence it is anti-symmetric. For J = 3/2 , the quark spins are aligned,

- $J_z = 3/2, \psi_{spin} = +3/2 = |\uparrow\uparrow\uparrow\rangle$
- $J_z = 1/2, \psi_{spin} = +1/2 = 1/\sqrt{3}(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)$
- $J_z = 1/2, \psi_{spin} = +1/2 = 1/\sqrt{3}(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle)$
- $J_z = -3/2, \psi = -3/2 = |\downarrow\downarrow\downarrow\rangle$

These wave functions are totally symmetric under quark exchange.

Flavour wave function

They are totally symmetric under quark exchange(two quarks)
for Δ baryons ($\Delta^+ = uud\rangle$)

$\psi^{\Delta^+}_{flavour} = |uud\rangle$

So, for the full decuplet

- $\Delta^{+++} = uuu$
- $\Delta^+ = 1/\sqrt{3}(uud + udu + duu)$
- $\Delta^0 = 1/\sqrt{3}(udd + dud + udd)$
- $\Delta^- = ddd$

Therefore, the total wave function will be:-

$\psi^\Delta_{total} = \psi^\Delta_{flavour} \otimes \psi^\Delta_{spin} \otimes \psi^\Delta_{color}$

10 Isospin Conservation

We are now going to prove how isospin is conserved due to strong interaction. I_3 is conserved by both the strong and electromagnetic interactions since the net number of u and d quarks so not change. Isospin is a mixture of u and d flavour symmetry with the consequences of u and d mass degeneracy, which we are going to talk about a little later.

We begin our investigation by looking at the upper spin states of the delta baryon of state Δ^0. We can construct its wave function using the upper two spin states ψ_{spin}:-

- $\Delta^0 = J_z = 3/2 = ddu, |\uparrow\uparrow\uparrow\rangle$
- $\Delta^0 = J_z = 1/2 = ddu, 1/\sqrt{3}(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)$

If we compare this with other zero charge states like Neutron and Σ^0 $(J_z = 1/2)$

- $N^0 = J_z = 1/2 = ddu, 1/\sqrt{6}(-2|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)$
- $\Sigma_0 = uds, 1/\sqrt{6}(-2|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)$

For Δ^0 and N^0, the coefficients of $|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle$ are equal since they are symmetric under the exchange of the first two quarks, which have identical flavours (dd). This however is not the case for the uds baryon , (strange component) so there must exist another state with isospin 0. [3/2 -1/2-1 =0, due to mixed symmetry and also when two quarks are swapped. The same thing happens when we consider positive charge states. There has to be an extra state with isospin 1]

Hence, the Λ^0 baryon

$\Lambda^0 = uds, 1/\sqrt{(2)}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle)$

Now, we know that the 3rd component of isospin is the eigenvalue operator which differentiates members of an isospin multiplet based on their quark content.

If the mass eigenstate is a solution to the Hamiltonian of a system and the mass eigenstates are also the isospin eigenstates it means that the strong interaction does not mix different isospin states as seen in the example above. (They have different isospin states 3/2, 1/2 and 0) (u and d have nearly the same mass) Each particle maintains a definite isospin value and what do we learn from this near mass degeneracy ?. **Strong interaction conserves isospin.**

So far, we have understood flavour dynamics and the flavour symmetry that emerges when we conserve a few quantum numbers. Just like isospin is one quantum number whose conservation is necessary for strong interactions. Strangeness conservation is also necessary to form flavour symmetry.

After the war, physicists noticed some of the particles produced had a ”strange” quality to it. As mentioned in the meson pairings section, these particles that were knocked off, occurred in pairs. When high-energy proton collision occurred, they produced kaons, lambda baryon, positively charged pions and protons(S= 0).

- $p^+ + \pi^- \rightarrow K^0 + \Lambda^0$
- $p^+ + \pi^- \rightarrow \Sigma^- + K^+$

Gell-mann suggested a new quantum number ”strangeness” charge must exist and therefore, needs to be conserved.

$S(p, n, \pi) = 0, \; S(K, \Lambda, \Sigma) = + - 1$

Now, to understand how particles are sorted into groups of mesons or baryons, we have to understand how they are further separated on the basis of their parity, strangeness and charge. Symmetrical patterns start to appear when these particles have their strangeness plotted against their charge. This strangeness symmetry is also an underlying symmetry of strong interaction.

All of the hadron nomenclature can be categorized in an eightfold way. Think about this as a periodic table but for particles. The mesons can be arranged in multiplets. The pseudoscalar mesons forms an octet $J^p = 0^-$

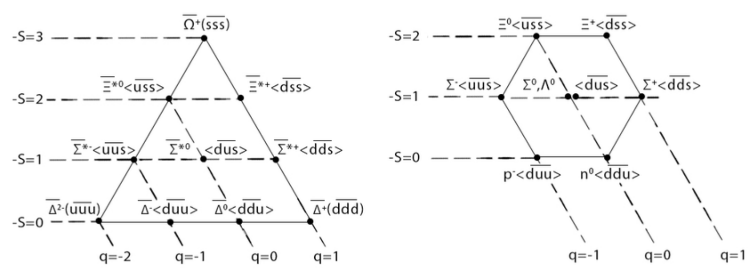

Figure 9: baryon decuplet(3/2) on the left and 1/2 baryon octet on the right. Eightfold representation, Source:- Researchgate

Similarly, the vector mesons ($J^p = 1^-$) form a nonet. In the next part, we will take a field theory approach to SU(3) symmetry and further our investigation into non-perturbative QCD.

11 References

- High energy physics.com, flavour symmetry
- Feynman’s lectures on strong interaction

- Dr physicsa - Youtube channel.
- QCD matter- wiki.
- Hadrons -wiki.