

$\mathcal{N} = 2$ Supersymmetry Algebra and BPS Bounds

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1 The Supersymmetry Algebra

For $\mathcal{N} = 2$ supersymmetry, the derivation of supercharges is naturally understood as an extension of the Poincaré algebra. The Poincaré algebra $SO(1, 3)$ is generated by translations P_μ ($\mu = 0, 1, 2, 3$) and the Lorentz generators $M_{\mu\nu} = -M_{\nu\mu}$. The supercharge Q , rather than being introduced by hand, poses a much more fundamental question—what is the largest possible continuous spacetime symmetry algebra that still contains the Poincaré algebra and is compatible with quantum mechanics? The answer is given by the Haag–Lopuszanski–Sohnius theorem (Haag et al., 1975), which extends the Coleman–Mandula theorem by allowing fermionic generators. Consequently, beyond the bosonic Poincaré generators $P_\mu, M_{\mu\nu}$, we introduce the fermionic generators $Q_\alpha^I, \bar{Q}_{\dot{\alpha}I}$, where $I = 1, 2$ labels the two independent supersymmetries (the $SU(2)_R$ index), while $\alpha, \dot{\alpha} = 1, 2$ are the Weyl spinor indices. Since these generators carry spinor indices, they transform in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group rather than as scalars or vectors.

1.1 Commutation with Translations

The first step is to determine how these generators transform under the translations of spacetime. Since translations commute among themselves, $[P_\mu, P_\nu] = 0$, and since supersymmetry should represent an internal extension of spacetime rather than introducing a preferred point, consistency requires the supercharges to commute with translations:

$$[P_\mu, Q_\alpha^I] = 0, \tag{1}$$

$$[P_\mu, \bar{Q}_{\dot{\alpha}I}] = 0. \tag{2}$$

Thus, the supercharges are conserved quantities whose values are independent of the spacetime positions. They may therefore be interpreted as global fermionic charges obtained by integrating conserved supercurrents.

1.2 Lorentz Transformation Properties

The Lorentz transformation properties follow directly from the nature of the spinor generators. The Lorentz generators satisfy

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu), \tag{3}$$

and acting on a Weyl spinor produces

$$[M_{\mu\nu}, Q_\alpha^I] = (\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I, \tag{4}$$

$$[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}I}] = (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}} \bar{Q}_{\dot{\beta}I}. \tag{5}$$

These relations uniquely identify the supercharges as Lorentz spinors (rather than ordinary bosonic generators). The $SU(2)_R$ index I remains untraced because it labels an internal symmetry independent of spacetime.

1.3 Fermionic Anticommutation Relations

Having fixed the bosonic commutators, the next task is to determine the fermionic anticommutation relations. Since the supercharges are Grassmann-odd operators, the graded Lie algebra requires their products to appear through anticommutators rather than commutators. The most general Lorentz-covariant ansatz is:

$$\{Q_\alpha^I, \bar{Q}_{\dot{\beta}J}\} = A \delta_J^I (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \quad (6)$$

where A is an undetermined normalization constant. Lorentz covariance leaves essentially no other vector operator available: the combination $(\sigma^\mu)_{\alpha\dot{\beta}}$ is the unique object that can convert one left-handed and one right-handed spinor into a spacetime vector.

The normalization constant is determined by the positivity of the Hamiltonian. Conventionally, one chooses $A = 2$, giving the familiar relation

$$\boxed{\{Q_\alpha^I, \bar{Q}_{\dot{\beta}J}\} = 2 \delta_J^I (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu.} \quad (7)$$

This equation is the central signature of supersymmetry. Two successive supersymmetry transformations do not generate another supersymmetry transformation but instead generate an ordinary spacetime translation. Supersymmetry can therefore be described as the “square root” of translations.

1.4 Central Charges

The anticommutators between supercharges of the same chirality require something more than what Lorentz covariance alone allows:

$$\{Q_\alpha^I, Q_\beta^J\} = \varepsilon_{\alpha\beta} Z^{IJ}, \quad (8)$$

where Z^{IJ} is the *central charge*, meaning that it commutes with all the other elements of the algebra. The exact nature of these central charges depends on the precise theory under consideration, but they must be constructed from other conserved quantities at hand. Z^{IJ} is antisymmetric in the R -symmetry indices:

$$Z^{IJ} = -Z^{JI}. \quad (9)$$

For $\mathcal{N} = 2$, antisymmetry implies that only one independent complex central charge can exist:

$$Z^{IJ} = \varepsilon^{IJ} Z. \quad (10)$$

Similarly,

$$\{\bar{Q}_{\dot{\alpha}I}, \bar{Q}_{\dot{\beta}J}\} = \varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon_{IJ} Z^\dagger. \quad (11)$$

These central charges commute with every generator of the algebra and play an essential role in the existence of BPS states. Their appearance is not arbitrary but follows from the graded Jacobi identities. One systematically inserts the proposed algebra into identities such as:

$$[M, \{a, a\}] = \{[M, a], a\} + \{a, [M, a]\}, \quad (12)$$

$$[P, \{a, a\}] = \{[P, a], a\} + \{a, [P, a]\}, \quad (13)$$

requiring every identity to hold. The Jacobi identity eliminates all Lorentz-noncovariant possibilities and forces any additional bosonic operator on the right-hand side of $\{a, a\}$ to commute with the entire Poincaré algebra. Such operators are precisely the central charges.

2 The Complete $\mathcal{N} = 2$ Supersymmetry Algebra

The complete $\mathcal{N} = 2$ supersymmetry algebra takes the form:

$$[P_\mu, P_\nu] = 0, \quad (14)$$

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu), \quad (15)$$

$$[M_{\mu\nu}, Q_\alpha^I] = (\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I, \quad (16)$$

$$[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}I}] = (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}} \bar{Q}_{\dot{\beta}I}, \quad (17)$$

$$[P_\mu, Q_\alpha^I] = [P_\mu, \bar{Q}_{\dot{\alpha}I}] = 0, \quad (18)$$

$$\{Q_\alpha^I, \bar{Q}_{\dot{\beta}J}\} = 2\delta_J^I (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \quad (19)$$

$$\{Q_\alpha^I, Q_\beta^J\} = \varepsilon_{\alpha\beta} \varepsilon^{IJ} Z, \quad (20)$$

together with the Hermitian conjugate relation for $\bar{Q}$. The structure of this algebra and its physical consequences are discussed in detail in Wess and Bagger (1992) and Weinberg (2000).

Analytically, these bosonic generators acquire a concrete realization in field theory through Noether's theorem. Starting from an $\mathcal{N} = 2$ -invariant Lagrangian, one can derive the conserved supercurrents $J_{\alpha I}^\mu$, satisfying

$$\partial_\mu J_{\alpha I}^\mu = 0. \quad (21)$$

Conserved supercharges can then be obtained as spatial integrals:

$$Q_\alpha^I = \int d^3x J_{\alpha I}^0(x), \quad \bar{Q}_{\dot{\alpha}I} = \int d^3x \bar{J}_{\dot{\alpha}I}^0(x). \quad (22)$$

Canonical quantization of the fields reproduces exactly the graded algebra defined above. Thus, the supercharges are not postulated ad hoc; they emerge as conserved Noether charges associated with fermionic supersymmetry symmetries, and their algebra is uniquely fixed by the graded Jacobi identities and the requirement that the extended symmetry closes on the Poincaré generators.

3 BPS Bounds

As noted previously, the anticommutators between supercharges of the same chirality introduce central charges. Our goal here is to understand the representations of this algebra, in conjunction with the original supersymmetry algebra. In the rest frame of a particle, $P_\mu = (m, 0, 0, 0)$, the supersymmetry algebra from Eq. (7) becomes:

$$\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}J}\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_J^I = 2m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta_J^I. \quad (23)$$

We illustrate the story with $\mathcal{N} = 2$ supersymmetry, although the general idea holds for any theory with extended supersymmetry. As we have seen, the antisymmetric central charge is necessarily just a complex number Z :

$$Z^{IJ} = 2\varepsilon^{IJ} Z. \quad (24)$$

For simplicity, we take Z to be real. We then define the following combinations of creation and annihilation operators:

$$a_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1^1 + \bar{Q}_2^2 \\ Q_2^1 - \bar{Q}_1^2 \end{pmatrix}, \quad b_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1^1 - \bar{Q}_2^2 \\ Q_2^1 + \bar{Q}_1^2 \end{pmatrix}. \quad (25)$$

Their anticommutation relations read:

$$\{a_\alpha, a_\beta^\dagger\} = 2(m + Z) \delta_{\alpha\beta}, \quad (26)$$

$$\{b_\alpha, b_\beta^\dagger\} = 2(m - Z) \delta_{\alpha\beta}, \quad (27)$$

with all other anticommutators vanishing. The quantities $\{a_\alpha, a_\beta^\dagger\}$ and $\{b_\alpha, b_\beta^\dagger\}$ are both positive-definite. Hence the corresponding right-hand sides must be non-negative as well. This is only true if the masses are bound by the central charges, i.e.

$$\boxed{m \geq |Z|}. \quad (28)$$

This also holds if Z is complex; one simply redefines the operators a and b using a phase to derive the same result. (Tong, 2021)

This formula is remarkable. Even before considering a specific example, recall that the central charge Z is some combination of conserved charges in quantum field theory. We learn that the masses of particles are bounded by their charges. This is known as the *BPS bound* (Witten and Olive, 1978).

3.1 Representation Theory: Long and Short Multiplets

The representation theory of this algebra depends crucially on whether $m > |Z|$ or $m = |Z|$.

Long multiplets. If $m > |Z|$, both $a_\alpha^\dagger$ and $b_\beta^\dagger$ act as creation operators. The result is a multiplet comprising 16 states. This is known as a *long multiplet*. More generally, for $\mathcal{N}$ supersymmetries, long multiplets contain $2^{2\mathcal{N}}$ states.

Short multiplets. What is more interesting is what happens when $m = |Z|$. In this case, half of the creation operators do not create anything. For example, when $m = Z$, the b_α operators must vanish on all states in the multiplet. Only $a_\alpha^\dagger$ act as creation operators. The result is a vector multiplet with 8 states, but now with mass $m = Z$. This is known as a *short multiplet*.

Therefore, BPS states can be defined as the states that saturate a lower bound on their energy derived from the supersymmetry algebra and, as a consequence, they belong to shortened (smaller) multiplets that are protected against quantum corrections.

4 Derivation of the BPS Bound

We now present a more systematic derivation of the BPS bound. Consider a state $|\Psi\rangle$ of mass M and momentum P_μ , with $P_\mu = (M, 0, 0, 0)$. Acting on such states, the supersymmetry algebra from Eq. (7) gives:

$$\{Q_\alpha^I, \bar{Q}_\beta^J\} = 2 \sigma_\alpha^\mu P_\mu \delta_{\beta}^J. \quad (29)$$

In order to derive the bound, we construct a positive operator Q , where I indexes a unit component vector:

$$Q \equiv \varepsilon_I \bar{Q}_\alpha^I \cdot v^{\dot{\alpha}}, \quad (30)$$

where $v^{\dot{\alpha}}$ is a unit two-component spinor ($v^{\dot{\alpha}} v_{\dot{\alpha}} = 1$). The expectation value is positive-definite:

$$\langle \Psi | Q^\dagger Q | \Psi \rangle \geq 0. \quad (31)$$

The expectation value must be non-negative by the positivity of the Hilbert space inner product. Evaluating this expectation value using Eq. (7) and $\{Q_\alpha^I, Q_\beta^J\} = \varepsilon_{\alpha\beta} \varepsilon^{IJ} Z$, and optimizing over the auxiliary vectors ε_I and v^α , yields the BPS inequality.

4.1 Systematic Derivation via a Positive-Definite Matrix

A more systematic derivation proceeds as follows. Define the positive-definite matrix:

$$M_J^{I\dot{\alpha}} = \langle \Psi | Q_\alpha^I \bar{Q}^{\dot{\alpha}J} | \Psi \rangle. \quad (32)$$

Using the algebra, this becomes:

$$M_J^{I\dot{\alpha}} = 2M \delta_J^{I\dot{\alpha}} - \langle \Psi | Q_\alpha^I \bar{Q}^{\dot{\alpha}J} | \Psi \rangle. \quad (33)$$

The second term is non-negative by the positivity of the norm. The central charge enters through the anticommutation relations $\{Q_\alpha^I, Q_\beta^J\} = \varepsilon_{\alpha\beta} \varepsilon^{IJ} Z$.

The complete analysis (Weinberg, 2000, Section 25.5) gives the bound:

$$M^2 \geq |Z|^2. \quad (34)$$

Taking the square root (with $M > 0$) for a physical state, we obtain:

$$\boxed{M \geq |Z|}. \quad (35)$$

This is the BPS bound, thus derived.

5 Classification of BPS States

As mentioned above, a state $|\Psi\rangle$ is called a BPS state if it saturates the BPS bound $M = |Z|$. Such a BPS state must satisfy $Q |\text{BPS}\rangle = 0$ for the choice of auxiliary vectors that optimize the inequality. This means that a subset of the supercharges annihilate the BPS state. The number of annihilated supercharges depends on the type of bound saturated, leading to the following classification:

- $\frac{1}{2}$ **BPS** (as discussed previously): Exactly half the supercharges annihilate the state. In $\mathcal{N} = 2$ with 8 supercharges, 4 are preserved and 4 are broken. The resulting supermultiplet has half the number of states of a generic (long) multiplet.
- $\frac{1}{4}$ **BPS**: Only a quarter of the supercharges annihilate the state. This occurs in extended supersymmetry ($\mathcal{N} \geq 4$) or for special configurations where multiple central charges are simultaneously saturated.
- $\frac{1}{2}$ **BPS objects of co-dimension 2**: (Nandi, 2025) A cosmic string (topological defect) is a $\frac{1}{2}$ BPS object. It preserves 4 of the 8 supercharges. The broken 4 supercharges give rise to 4 fermionic zero-modes on the string worldsheet, which then reorganize into 2 left-moving and 2 right-moving worldsheet fermions of the $\mathcal{N} = (2, 2)$ algebra in 1+1 dimensions.

Proposition 5.1 (Short multiplet). *A $\frac{1}{2}$ BPS state in $\mathcal{N} = 2$ supersymmetry belongs to a short supermultiplet of dimension half that of a long multiplet, and its mass (or, for an extended object, its tension) is exactly equal to the magnitude of the central charge: $T = |Z|$.*

Proof. When $M = |Z|$, the positive operator $Q^\dagger Q$ has zero eigenvalues, meaning that there exist states annihilated by Q . The annihilated supercharges pair bosonic and fermionic states within the multiplet, halving the number of independent states. For an extended object such as a cosmic string, the tension T (energy per unit length) plays the role of mass in the bound, giving $T = |Z|$ for a BPS cosmic string. $\square$

6 BPS States, the Bogomol'nyi Equations, and Cosmic Strings

The connection between BPS states and cosmic strings arises naturally in supersymmetric gauge theories containing spontaneously broken $U(1)$ symmetries (Nielsen and Olesen, 1973; Shifman and Yung, 2009). When a complex scalar field acquires a non-zero vacuum expectation value, the vacuum manifold becomes topologically non-trivial. In the abelian Higgs model, the vacuum satisfies

$$|\Phi| = v, \quad (36)$$

so that the vacuum manifold is the circle S^1 . Since the first homotopy group satisfies

$$\pi_1(S^1) = \mathbb{Z}, \quad (37)$$

field configurations can possess an integer winding number that cannot be continuously deformed to the trivial vacuum. These topologically stable configurations are the Abrikosov–Nielsen–Olesen (ANO) vortices (Nielsen and Olesen, 1973). Originally discovered in the context of superconductivity, they were later recognized as the prototype of cosmic strings in relativistic field theory. If such vortices are produced during a cosmological phase transition in the early universe, they may stretch over cosmological distances and are then identified as cosmic strings.

6.1 The Abelian Higgs Model and Vortex Tension

The energy per unit length (or tension) of an ANO vortex is obtained from the Abelian Higgs Lagrangian:

$$T = \int d^2x \left[|D_i \phi|^2 + \frac{1}{2} B^2 + \frac{\beta}{4} (|\phi|^2 - v^2)^2 \right]. \quad (38)$$

We know that the vacuum manifold forms a circle S^1 because $\pi_1(S^1) = \mathbb{Z}$. This phase cannot be unwound continuously. Instead, the scalar field winds around the circle:

$$\phi(r, \theta) = v f(r) e^{in\theta}, \quad (39)$$

where the integer $n = 0, \pm 1, \pm 2, \dots$ is the winding number. This topological obstruction produces a stable string.

In the Lagrangian (38), D_i is the gauge-covariant derivative,

$$D_\mu = \partial_\mu - ieA_\mu, \quad (40)$$

where ∂_μ is the ordinary spacetime derivative, e is the electric charge, and A_μ is the $U(1)$ gauge field. The covariant derivative D_μ transforms covariantly under local gauge transformations. For static vortices, we only need the spatial directions:

$$D_i = \partial_i - ieA_i, \quad i = 1, 2. \quad (41)$$

Hence D_1 acts along the x -direction and D_2 acts along the y -direction.

6.2 The Bogomol'nyi Decomposition

At the critical coupling $\beta = e^2$, the energy functional can be reorganized by completing the square, leading to the Bogomol'nyi decomposition (Bogomol'nyi, 1976; Prasad and Sommerfield, 1975):

$$T = \int d^2x \left[|D_1 \phi \pm iD_2 \phi|^2 + \frac{1}{2} (B \mp e(|\phi|^2 - v^2))^2 \right] + 2\pi v^2 |n|. \quad (42)$$

Since every squared term is non-negative,

$$T \geq 2\pi v^2 |n|. \quad (43)$$

The lower bound depends only on the winding number, making it a topological quantity rather than one sensitive to local field fluctuations. The reason the BPS equation contains $D_1 \pm iD_2$ is that the vortex is a two-dimensional object lying in the x - y plane. The lower bound is insensitive to smooth local deformations of the fields.

The minimum tension is reached precisely when the squared terms vanish, yielding the first-order Bogomol'nyi (BPS) equations:

$$(D_1 \pm iD_2)\phi = 0, \quad (44)$$

$$B = \pm e(v^2 - |\phi|^2). \quad (45)$$

6.3 Magnetic Flux and Vortex Structure

The gauge field produces a magnetic field through its field strength. For a static vortex, there is only one nonzero component of the magnetic field, $B = F_{12}$. Explicitly,

$$B = \partial_1 A_2 - \partial_2 A_1. \quad (46)$$

Since the vortex extends along the z -axis, the magnetic field points along z , while the fields themselves depend only on x and y . Physically, the vortex maps to the magnetic flux confined inside a narrow tube.

In an $\mathcal{N} = 2$ supersymmetric gauge theory (Seiberg and Witten, 1994; Shifman and Yung, 2009), the above vortex solutions preserve one-half of the supersymmetry generators and therefore constitute $\frac{1}{2}$ BPS strings. Their tension exactly saturates the supersymmetry bound:

$$T = |Z|, \quad (47)$$

where Z is the appropriate topological central charge appearing in the extended supersymmetry algebra. Consequently, the string tension is now protected against quantum corrections, making BPS cosmic strings among the few non-perturbative objects whose physical properties can be determined exactly.

From a broader perspective, ANO vortices provide the field-theoretic realization of the same algebraic structure that governs BPS particles such as monopoles and dyons (Witten and Olive, 1978). The only difference is that the conserved quantity is no longer the mass of a point-particle but the tension of an extended one-dimensional object. In both cases, supersymmetry relates the energy of the configuration directly to a topological charge, explaining the stability of these solutions. This connection between topology, gauge symmetry, and supersymmetry makes BPS cosmic strings an important bridge between supersymmetric quantum mechanics, cosmology, and string theory.

7 BPS States and Extremal Black Holes

A profound application of BPS states appears in supersymmetric gravity (Ferrara et al., 1995). This is the specific example that was mentioned in Section 6: here the BPS bound becomes a relation between the mass and conserved charges of a black hole. In ordinary Einstein–Maxwell theory, a charged black hole is described by the Reissner–Nordström solution, whose mass M and electric and magnetic charges satisfy

$$M^2 \geq q_e^2 + q_m^2. \quad (48)$$

The limiting case

$$M^2 = q_e^2 + q_m^2 \quad (49)$$

defines an *extremal black hole*, characterized by zero Hawking temperature and the minimum possible mass for a given charge. Supersymmetry provides an elegant explanation for this condition: in an $\mathcal{N} = 2$ supergravity theory, the supersymmetry algebra contains a central charge Z , and positivity of the supercharge anticommutators once again implies the BPS inequality:

$$M \geq |Z|. \quad (50)$$

An extremal supersymmetric black hole is precisely one that saturates this bound,

$$M = |Z|, \quad (51)$$

so that it preserves a fraction of the original supersymmetry and becomes a BPS black hole.

7.1 Physical Consequences of the BPS Property

The BPS property has many remarkable physical consequences. Because part of the supersymmetry remains unbroken, the black hole belongs to a short supermultiplet whose mass is exactly protected against quantum corrections. Furthermore, the near-horizon geometry of many $\mathcal{N} = 2$ BPS black holes exhibits the universal structure:

$$\text{AdS}_2 \times S^2. \quad (52)$$

The scalar fields evolve, flowing from arbitrary values at spatial infinity to fixed values at the horizon determined solely by the conserved electric and magnetic charges. This *attractor mechanism* (Ferrara et al., 1995) is a hallmark of BPS black holes in supergravity: the horizon values of the scalars are independent of their asymptotic boundary conditions and depend only on the charges.

This universality allows quantities such as the horizon area $A = 4\pi r_H^2$ and the Bekenstein–Hawking entropy

$$S_{\text{BH}} = \frac{A}{4G} \quad (53)$$

to be expressed entirely in terms of charge invariants. Because the mass, charges, and entropy are all protected by supersymmetry, BPS black holes provide one of the few settings in which exact non-perturbative comparisons between supergravity and string theory become possible (Seiberg and Witten, 1994). They occupy a central position in current studies of black hole microstates, dualities, and the microscopic origin of black hole entropy.

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